

# Self-tuning and de Sitter brane intersections in 6-dimensional brane models

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**Abstract.** We study the self-tuning of general brane junctions and brane networks on 6-dimensional space-time. For general brane junctions, there may exist one fine-tuning among the brane tensions. For the brane networks, similar to the 5-dimensional self-tuning brane models, the brane tensions can be set arbitrarily and there exists a singularity for the metric and bulk scalar. If we want to regularize the singularity, we will introduce fine-tuning among the brane tensions. In addition, because the 4-dimensional cosmological constant we observe may be positive and very small, we discuss the brane network with de Sitter brane intersections by introducing a bulk scalar.

## 1 Introduction

There are some unattractive features in the standard model (SM) which may imply new physics, although the SM is very successful as seen from the experiments at LEP and Tevatron. One of these problems is that the gauge interactions and gravitational interaction are not unified. Another is the gauge hierarchy problem. As we know, several solutions to the gauge hierarchy problem have been proposed: technicolor and compositeness, which lacks calculability; weak-scale supersymmetry which was the leading candidate for the extension of the standard model several years ago; and conformality, which is similar in spirit to supersymmetry: one just replaces one symmetry (supersymmetry) with another (conformal symmetry) above the TeV scale, and both approaches predict new physics at the TeV scale [1].

About three years ago, it was suggested that large compactified extra dimensions may also be a solution to the gauge hierarchy problem [2], because a low  $(4+n)$ -dimensional Planck scale ( $M_X$ ) may result in a large 4-dimensional Planck scale ( $M_{P1}$ ) due to the large physical volume ( $V_p^n$ ) of the extra dimensions:  $M_{P1}^2 = M_X^{2+n} V_p^n$ . In addition, Randall and Sundrum [3] proposed another scenario: that the extra dimension is an orbifold, and the size of the extra dimension is not large but the 4-dimensional mass scale in the standard model is suppressed by an exponential factor from the 5-dimensional mass scale due to the

exponential warp factor. Furthermore, they suggested that the fifth dimension might be coordinate non-compact [4], and there may exist only one brane with a positive tension at the origin; however, there still exists the gauge hierarchy problem. The remarkable aspect of the second scenario is that it gives rise to a localized graviton field. Afterwards, a lot of 5-dimensional models with 3-branes were built [5, 6], and models with co-dimension one brane(s) were constructed on a 6-dimensional and higher-dimensional space-time [7–11].

In the above model building, all the models with a warp factor in the metric have a negative bulk cosmological constant. However, in string theory, it is natural to take the bulk cosmological constant to be zero since the tree-level vacuum energy in the generic critical closed string compactifications (supersymmetric or not) vanishes. And a zero bulk cosmological constant is natural in the scenario in which the bulk is supersymmetric (though the brane need not be), or the quantum corrections to the bulk are small enough to be neglected in a controlled expansion. So, how to construct models with zero bulk cosmological constant is an interesting question in the model building, because such kinds of models are still interesting if the bulk corrections to the bulk cosmological constant  $\Lambda$  were very small, which can happen for instance if the supersymmetry breaking is localized in a small neighborhood of the branes, or if the supersymmetry breaking scale in the bulk is small enough. Moreover, if all the gauge fields and matter fields were confined to the branes, the quantum corrections of these fields to the brane tensions might not affect the models with  $\Lambda = 0$ .

One scenario is that we introduce not only a space-like extra dimension, but also a time-like extra dimension [10]. The good aspect of this approach is that there is

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no singularity; however, there exists fine-tuning and one might have the problems arising from the time-like extra dimension: unitarity and causality. Another scenario was proposed where a scalar  $\phi$ , which does not have a bulk potential, is introduced [12]. In the second scenario,  $\phi$  becomes singular at a finite distance along the extra dimension and the warp factor in the metric vanishes at the singularity. The good aspect of this approach is that the brane tension can be set arbitrarily. However, the  $Z_2$  symmetric and 4-dimensional Poincaré invariant solution is unstable under the bulk quantum corrections, and any procedure which regularizes the singularity will introduce the fine-tuning which the self-tuning is supposed to avoid [13]. Furthermore, a simple no-go theorem [14] relating the self-tuning solutions to the cosmological constant for observers on the brane, which relies on a singularity in an extra dimension, shows that it is impossible to shield the singularity from the brane by a horizon [15], unless the positive energy condition is violated in the bulk or on the brane, or the 3-brane has a spatial curvature.

In this paper, we would like to discuss the self-tuning of general brane junctions [11], a simple brane intersection, and a brane network [9] on a 6-dimensional space-time by introducing a bulk scalar without bulk potential. For general brane junction models, there may exist one fine-tuning among the total brane tensions<sup>1</sup>. However, in the brane intersection models or brane networks where the branes are co-dimension one hypersurfaces, the brane tensions can be set arbitrarily, because the constraint in the brane junctions is satisfied automatically. Similar to the 5-dimensional brane models, there exists the singularity for the metric and bulk scalar in the self-tuning of brane networks. If we want to regularize the singularity, for example, we require the extra dimensions to be compact and introduce cut-off branes; then we will have fine-tuning among the brane tensions.

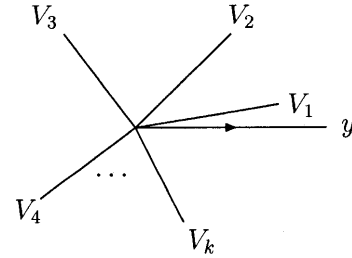
In addition, as we know, our universe may have a very small positive cosmological constant, so we discuss a brane network with de Sitter brane intersections. Suppose we have  $n$  space-like extra dimensions whose coordinates are  $y^i$ , where  $i = 1, 2, \dots, n$ , and the branes are co-dimension one hypersurfaces which are determined by the algebraic equations  $y^i = r$ , where  $r$  is a real number. If one assumed the metric

$$ds^2 = \Omega^{-2} \left( -dt^2 + \sum_{i=1}^3 e^{2Ht} dx^i dx^i + \sum_{i=1}^n dy^{i2} \right), \quad (1)$$

it is not difficult to show that there does not exist a solution for  $n > 1$ <sup>2</sup>, i.e., we do not have such a kind of brane networks with de Sitter brane intersections. In order to obtain the solutions, we introduce a bulk scalar which has a bulk potential; in other words, we add one degree of freedom to the system. We present a 6-dimensional brane network with three (two) 4-branes whose extra dimension

<sup>1</sup> Although it is not essentially self-tuning if there was one fine-tuning among the brane tensions, we still call it “self-tuning”, which means that the bulk potential for  $\phi$  is zero

<sup>2</sup> For the  $n = 1$  solution, for example, see [16]



**Fig. 1.** A junction (3-brane) of  $k$  semi-infinite 4-branes. The angular positions of branes are measured from the  $y$ -axis and the brane tensions are denoted by  $V_i$

coordinates are  $y$  and  $z$ , one along the  $y$  direction and two (one) along the  $z$  direction. The solution has no singularity and all the brane tensions have similar forms in terms of the scalar. Similarly, one can also discuss the general brane networks with de Sitter or anti-de Sitter brane intersections.

## 2 Self-tuning of the brane junctions and networks

First, let us discuss the self-tuning of the general brane junctions. The set-up is given in Fig. 1; we use the metric with signature  $(-, +, +, +, +, +)$ . The 6-dimensional gravitational action describing the system is

$$S = \frac{1}{2\kappa^2} \int d^4x dy dz \sqrt{-g} (R - \partial_A \phi \partial^A \phi) - \sum_{i=1}^k \int d^4x dy dz \sqrt{-g^{(i)}} V_i(\phi) \delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w}), \quad (2)$$

where  $\kappa^2 = M_X^{-4}$  is the 6-dimensional coupling constant of gravity,  $M_X$  is the 6-dimensional Planck scale, and  $R$  is the curvature scalar. The vectors

$$\begin{aligned} \vec{n}_i &= (-\sin \alpha_i, \cos \alpha_i), & \vec{l}_i &= (\cos \alpha_i, \sin \alpha_i) \\ & \text{and } \vec{w} &= (y, z) \end{aligned} \quad (3)$$

are defined so that  $\delta(\vec{n}_i \cdot \vec{w})$  is the line in the  $y$ - $z$  plane which contains the  $i$ th brane and  $\theta(\vec{l}_i \cdot \vec{w})$  serves to cut the irrelevant half of the line. Thus,  $\delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w})$  defines the location of the  $i$ th semi-infinite brane. The 5-dimensional metric on the  $i$ th half-brane is

$$g_{AB}^{(i)} \equiv g_{AB}(z = y \tan \alpha_i). \quad (4)$$

Assuming the metric to be conformally flat, it can be written as

$$ds^2 = \Omega^{-2} (\eta_{\mu\nu} dx^\mu dx^\nu + dy^2 + dz^2), \quad (5)$$

where  $\Omega \equiv \Omega(y, z)$ . Then the Einstein equations are

$$\begin{aligned} G_{AB} &= \kappa^2 T_{AB} = \partial_A \phi \partial_B \phi - \frac{1}{2} g_{AB} (\partial \phi)^2 \\ &- \kappa^2 \Omega^{-1} \sum_{i=1}^k \Gamma_{AB}^{(i)} V_i(\phi) \delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w}), \end{aligned} \quad (6)$$



four singular points along the two branes:  $(y = -e/a, z = 0)$ ,  $(y = e/a, z = 0)$ ,  $(y = -e/b \cos \alpha, z = -e/b \sin \alpha)$ , and  $(y = e/b \cos \alpha, z = e/b \sin \alpha)$ , we can draw four straight lines from  $(y = -e/a, z = 0)$  and  $(y = e/a, z = 0)$  to  $(y = -e/b \cos \alpha, z = -e/b \sin \alpha)$  and  $(y = e/b \cos \alpha, z = e/b \sin \alpha)$ . On those four lines,  $\phi$  is singular and  $\Omega^{-1}$  is zero.

Third, we discuss a brane network with four 4-branes on the extra space manifold  $R^1/Z_2 \times R^1/Z_2$ . Two branes along the  $y$  direction are located at  $y = 0$  and  $y = y_1$ , respectively, and two branes along the  $z$  direction are located at  $z = 0$  and  $z = z_1$ , respectively. The set-up is given in Fig. 3. The action for this model is

$$S = S_{\text{Bulk}} + S_{\text{Branes}}, \quad (22)$$

where

$$S_{\text{Bulk}} = \frac{1}{2\kappa^2} \int d^4x dy dz \sqrt{-g} (R - \partial_A \phi \partial^A \phi), \quad (23)$$

$$\begin{aligned} S_{\text{Brane}} = & - \int d^4x dy dz (\sqrt{-g^{(1)}} V_1(\phi) \delta(y) \\ & + \sqrt{-g^{(3)}} V_3(\phi) \delta(y - y_1)) \\ & - \int d^4x dy dz (\sqrt{-g^{(2)}} V_2(\phi) \delta(z) \\ & + \sqrt{-g^{(4)}} V_4(\phi) \delta(z - z_1)), \end{aligned} \quad (24)$$

where  $g^{(i)}$  for  $i = 1, 2, 3, 4$  is the metric on the  $i$ th brane, which can be obtained by restriction. The detailed calculation is similar, so we just give the result. Assuming the conformal metric

$$ds^2 = \Omega^{-2} (\eta_{\mu\nu} dx^\mu dx^\nu + dy^2 + dz^2), \quad (25)$$

we obtain the solution for the conformal factor and  $\phi$ :

$$\Omega = (a|y - y_1| + b|z - z_1| + cy + dz + e)^{-1/4}, \quad (26)$$

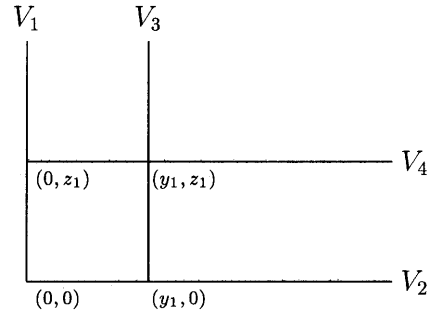
$$\phi = -2\sqrt{5} \ln (a|y - y_1| + b|z - z_1| + cy + dz + e)^{-1/4}, \quad (27)$$

and the brane tensions are

$$\begin{aligned} \kappa^2 V_1(\phi) &= -(c - a) e^{-(\sqrt{5}/2)\phi}, \\ \kappa^2 V_2(\phi) &= -(d - b) e^{-(\sqrt{5}/2)\phi}, \end{aligned} \quad (28)$$

$$\begin{aligned} \kappa^2 V_3(\phi) &= -2ae^{-(\sqrt{5}/2)\phi}, \\ \kappa^2 V_4(\phi) &= -2be^{-(\sqrt{5}/2)\phi}. \end{aligned} \quad (29)$$

So, if  $c > a$ ,  $d > b$  and  $e > 0$ , the metric and  $\phi$  do not have a singularity or vanish at finite distance from the origin, but they are divergent at infinity. The brane tensions can be set arbitrarily, for instance, if  $a < 0$  and  $b < 0$ , the brane tensions  $V_3$  and  $V_4$  are positive, and the brane tensions  $V_1$  and  $V_2$  are negative. In order to avoid the divergence in the metric and  $\phi$ , we can introduce two cut-off 4-branes:  $V_5$  which is located at  $y = y_2$ , and  $V_6$  which is located at  $z = z_2$ , where  $y_2 > y_1$  and  $z_2 > z_1$ . So,



**Fig. 3.** A network of four 4-branes on the 6-dimensional space-time  $M^4 \times R^1/Z_2 \times R^1/Z_2$

the extra space manifold is  $S^1/Z_2 \times S^1/Z_2$ . Because the tensions for the cut-off 4-branes are

$$\begin{aligned} \kappa^2 V_5(\phi) &= (c + a) e^{-(\sqrt{5}/2)\phi}, \\ \kappa^2 V_6(\phi) &= (d + b) e^{-(\sqrt{5}/2)\phi}, \end{aligned} \quad (30)$$

we will have fine-tuning among the brane tensions, which is similar to the 5-dimensional self-tuning models [13].

### 3 Brane network with de Sitter brane intersection

Because the 4-dimensional cosmological constant we observe is positive although it is very small, we would like to discuss the brane network with de Sitter brane intersections. In order to have the solutions, we introduce one bulk scalar  $\phi$  whose bulk potential does not vanish. Assume that we have three 4-branes, one along the  $y$  direction at  $y = 0$ , two along the  $z$  direction at  $z = 0$  and  $z = z_1$ . The set-up is given in Fig. 4b. The metrics on the branes can be obtained by the restriction

$$g_{AB}^{(1)} \equiv g_{AB}(y = 0), \quad \text{where } A, B \neq y, \quad (31)$$

$$g_{AB}^{(2)} \equiv g_{AB}(z = 0), \quad \text{where } A, B \neq z, \quad (32)$$

$$g_{AB}^{(3)} \equiv g_{AB}(z = z_1), \quad \text{where } A, B \neq z. \quad (33)$$

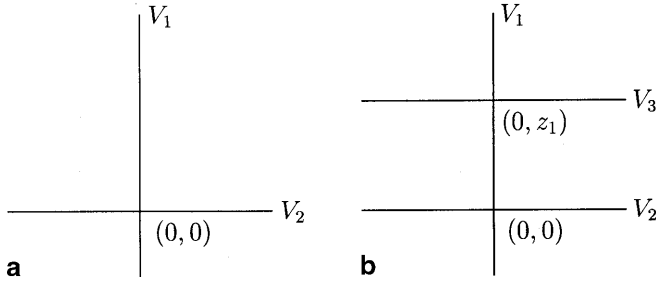
The action for this system is

$$S = S_{\text{Bulk}} + S_{\text{Branes}}, \quad (34)$$

where

$$S_{\text{Bulk}} = \int d^4x dy dz \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{2} \partial_A \phi \partial^A \phi - \Lambda(\phi) \right), \quad (35)$$

$$\begin{aligned} S_{\text{Brane}} = & - \int d^4x dy dz (\sqrt{-g^{(1)}} V_1(\phi) \delta(y) \\ & - \int d^4x dy dz (\sqrt{-g^{(2)}} V_2(\phi) \delta(z) \\ & + \sqrt{-g^{(3)}} V_3(\phi) \delta(z - z_1)). \end{aligned} \quad (36)$$



**Fig. 4.** **a** Two 4-branes with de Sitter brane intersection; **b** three 4-branes with two de Sitter brane intersections

With the following conformal metric:

$$ds^2 = \Omega^{-2} \left( -dt^2 + \sum_{i=1}^3 e^{2Ht} dx^i dx^i + dy^2 + dz^2 \right), \quad (37)$$

we obtain the Einstein equations

$$4 \frac{\partial^2 \Omega}{\partial y^2} = \Omega \left( \frac{\partial \phi}{\partial y} \right)^2 + V_1(\phi) \delta(y) + 3H^2 \Omega, \quad (38)$$

$$4 \frac{\partial^2 \Omega}{\partial z^2} = \Omega \left( \frac{\partial \phi}{\partial z} \right)^2 + V_2(\phi) \delta(z) + V_3(\phi) \delta(z - z_1) + 3H^2 \Omega, \quad (39)$$

$$20 \left( \frac{\partial \Omega}{\partial y} \right)^2 + 20 \left( \frac{\partial \Omega}{\partial z} \right)^2 = \Omega^2 \left( \frac{\partial \phi}{\partial y} \right)^2 + \Omega^2 \left( \frac{\partial \phi}{\partial z} \right)^2 - 2\Lambda(\phi) + 18H^2 \Omega^2, \quad (40)$$

$$4 \frac{\partial^2 \Omega}{\partial y \partial z} = \Omega \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial z}, \quad (41)$$

and the equation of motion for  $\phi$ ,

$$\begin{aligned} \Omega^2 \frac{\partial^2 \phi}{\partial z^2} + \Omega^2 \frac{\partial^2 \phi}{\partial y^2} = & +4\Omega \frac{\partial \Omega}{\partial y} \frac{\partial \phi}{\partial y} + 4\Omega \frac{\partial \Omega}{\partial z} \frac{\partial \phi}{\partial z} \\ & + \Omega \frac{\partial V_1(\phi)}{\partial \phi} \delta(y) + \Omega \frac{\partial V_2(\phi)}{\partial \phi} \delta(z) \\ & + \Omega \frac{\partial V_3(\phi)}{\partial \phi} \delta(z - z_1) + \frac{\partial \Lambda(\phi)}{\partial \phi}. \end{aligned} \quad (42)$$

If  $V_3(\phi) = 0$ , i.e., there are two 4-branes and the set-up is given in Fig. 4a. The conformal factor and  $\phi$  are

$$\Omega = \exp \left\{ \frac{3}{8} H^2 [(|y| + c_1)^2 + (|z| + c_2)^2 + c_3] \right\}, \quad (43)$$

$$\phi = \frac{3}{4} H^2 [(|y| + c_1)^2 + (|z| + c_2)^2 + c_3], \quad (44)$$

the bulk potential for  $\phi$  is

$$\Lambda(\phi) = 9 \left( 1 + \frac{1}{2} H^2 c_3 \right) H^2 e^\phi - 6H^2 \phi e^\phi, \quad (45)$$

and the brane tensions are

$$V_1(\phi) = 6e^{\phi/2}, \quad V_2(\phi) = 6e^{\phi/2}. \quad (46)$$

So both branes have positive tensions.

Now, we consider the case  $V_3(\phi) \neq 0$ , because we need at least three 4-branes to solve the gauge hierarchy problem. The conformal factor and  $\phi$  are

$$\Omega = \exp \left\{ \frac{3}{8} H^2 [(|y| + c_1)^2 + (|z| - |z - z_1| - z + c_2)^2 + c_3] \right\}, \quad (47)$$

$$\phi = \frac{3}{4} H^2 \left[ (|y| + c_1)^2 + (|z| - |z - z_1| - z + c_2)^2 + c_3 \right]; \quad (48)$$

the bulk potential for  $\phi$  is

$$\Lambda(\phi) = 9 \left( 1 + \frac{1}{2} H^2 c_3 \right) H^2 e^\phi - 6H^2 \phi e^\phi, \quad (49)$$

and the brane tensions are

$$V_1(\phi) = 6H^2 e^{\phi/2}, \quad V_2(\phi) = 6H^2 e^{\phi/2}, \quad (50)$$

$$V_3(\phi) = -6H^2 e^{\phi/2}. \quad (51)$$

Thus, the third brane has negative tension. By the way, all the brane tensions have similar forms in terms of the scalar.

Similarly, we can discuss the general brane networks with de Sitter or anti-de Sitter brane intersections.

## 4 Conclusion

We study the self-tuning of general brane junctions and brane networks on the 6-dimensional space-time. For the general brane junctions, there may exist one fine-tuning among the brane tensions. For the brane networks, similar to the 5-dimensional self-tuning brane models, the brane tensions can be set arbitrarily and there exists a singularity for the metric and bulk scalar. If we want to regularize the singularity, we will introduce the fine-tuning among the brane tensions. In addition, because the 4-dimensional cosmological constant we observe may be positive and very small, we discuss the brane network with de Sitter brane intersections by introducing a bulk scalar.

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